A SIMPLE SIMULATION OF ELECTRON-PROTON INSTABILITY

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Abstract

The electron-proton instability of a long, intense, and partially neutralized proton bunch is simulated by numerically solving the equations of motion for the centroid of the proton beam and for the macro-particles of the trapped electrons. The study takes into account the effects of variable line densities, the generation of the secondary electrons, and the multipacting of electrons. The results here are consistent and are qualitatively in good agreement with the earlier simulations using the centroid model and experimental observations. It is found that with only a few percent neutralization, the PSR beam can become unstable. It is also found that the enhancement of the instability due to the electron multiplication may occur after the oscillation of the proton beam has grown to large amplitude.

1 Introduction

Evidences have been collected to show that the instability in the PSR is an *e-p* instability.

The same kind of instability has been previously observed in the Bevatron and ISR.

The basic mechanism has been understood, the recent PSR upgrade as well as SNS and ESS are calling for more detailed understanding of the instability.

Earlier simulation program using the centroid model has recently been modified:

- replace the electron centroid by macro-electrons,

- include the secondary emission of electrons due to the impact of electrons on the beam pipe.

The emphasis here is to investigate the possibility of multipacting and the effect of electron multiplication on the instability.

2 The Model and the Numerical Approach

2.1 Model

A proton bunch of length L with a round cross-section of radius a, traveling with a constant speed v inside a perfect conducting pipe of radius b.

Linear transverse focusing on protons. Uniformly distribution of protons in the transverse direction.

The proton bunch is partially neutralized by electrons.

Use a Cartesian coordinate system: z-axis in the direction of proton propagation, y-axis perpendicular to the ring, and the origin at the center of the beam cross section.

Proton and electron line-densities, λ_p and λ_e, depend on z.

Assume the system is unstable in the y-direction only.

Neglect the axial motion of electrons and the synchrotron oscillation of protons.

Study the motion of the proton beam centroid $Y_p(z, t)$ = averaged displacement of protons at (z, t), where t = time.

The equation of motion for the centroid of protons is

$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial z}\right)^2 Y_p + \omega_\beta^2 Y_p = \frac{1}{\gamma}\sum_{j=1}^{N_e}\frac{F_{ej}}{m_p} - C_d\left(\frac{\partial Y_P}{\partial t} + v\frac{\partial Y_P}{\partial z}\right) ,$$

(1)

ω_β = betatron frequency due to the external focusing,
F_{ej} = force due to the jth electron,
$\gamma = (1 - v^2/c^2)^{-1/2}$,
c = speed of light,
m_p = rest mass of a proton,
C_d = damping constant .

The second term on the RHS is due to the damping of the coherent proton oscillation caused by the tune spread.

Neglecting the interaction among electrons, the equation of transverse motion for the jth electron at (y_{ej}, z, t) is

$$\frac{d^2 y_{ej}}{dt^2} = \frac{F_p(y_{ej}, z, t)}{m_{ej}} ,$$

(2)

$F_p(y_{ej}, z, t)$ = the force due to the proton beam,
m_{ej} = the mass of the jth electron.

2.2 Numerical Approach

Computations are carried out on the coordinate frame moving with the proton bunch.

Use macro-Protons and macro-electrons.

The proton bunch and the electron cloud are divided each into N slices (grid) each in the z-direction.

Each proton slice contains one macro-proton. The charges and the masses of macro-protons are assigned according to $\lambda_p(z)$.

Each electron slice contains n_e macro-electrons.
Electrons have two components:
the wall-electrons (from wall) n_{ew}, charge c_{ew}, mass m_{ew},
the core-electrons n_{ec}, charge c_{ec}, mass m_{ec},
$n_e = n_{ec} + n_{ew}$, and $c_e = c_{ec}n_{ec} + c_{ew}n_{ew}$.

The acceleration of the jth macro-electron due to the field of protons is approximated by

$$\frac{F_p(y_{ej}, z)}{m_{ej}} \approx$$

$$\begin{cases} -\dfrac{e^2 q_{ej}\lambda_p}{2\pi\epsilon_o m_{ej}}\left(\dfrac{Y_p}{b^2 - y_e Y_p} + \dfrac{y_e - Y_p}{a^2}\right), & \text{for } |y_e - Y_p| \leq a, \\[4mm] -\dfrac{e^2 q_{ej}\lambda_p}{2\pi\epsilon_o m_{ej}}\left(\dfrac{Y_p}{b^2 - y_e Y_p} + \dfrac{1}{y_e - Y_p}\right), & \text{for } |y_e - Y_p| \geq a, \end{cases}$$

$$\tag{3}$$

$e =$ unit charge,
$q_{ej} =$ total charge of the jth macro-electron,
$\epsilon_o =$ permittivity of the free space.

The force on a macro-proton due to the jth macro-electron is approximated by

$$F_{ej} \approx$$

$$\begin{cases} -\dfrac{e^2 q_{ej}\lambda_p}{2\pi\epsilon_o} \left(\dfrac{y_{ej}}{b^2 - y_{ej}Y_p} + \dfrac{Y_p - y_{ej}}{a_e^2} \right), & \text{for } |Y_p - y_{ej}| \le a_e, \\[4mm] -\dfrac{e^2 q_{ej}\lambda_p}{2\pi\epsilon_o} \left(\dfrac{y_{ej}}{b^2 - y_{ej}Y_p} + \dfrac{1}{Y_p - y_{ej}} \right), & \text{for } |Y_p - y_{ej}| \ge a_e, \end{cases}$$

$$(4)$$

a_e = "radius" of a macro-electron used to avoid singularity.

Eqs. (1) and (2) are solved by using the Runge-Kutta-Gill method.

Chose the time step $\Delta t = L/(vN)$.

In every Δt, all electron-slices are advanced by one grid toward the tail of the proton bunch to simulate the relative drifting of electrons; a new slice is created at the head of the proton bunch.

At every time step, use random number to select a slice in which two wall-electrons are created on the beam pipe to simulate the electron generation on the pipe due to the lost protons.

The total number of macro-particles = constant all the time.

Modeling the accumation of the electrons from gas scattering: a weight function $W_e(z)$ for the charge and the mass of macro-electrons is introduced so that

$$q_{ej} = c_{ej} W_e \quad .$$

(5)

If the electron generation per proton is a constant value, then roughly,

$$W_e(z) \propto \int_0^z \lambda_p(z') dz' \quad .$$

(6)

When an impact on wall is detected, the charge and the mass of the impinging macro-electron is adjusted (charge/mass = const.) according to the secondary emission yield (SEY).

The SEY is calculated using:

$$\delta_{ts}(E_0/\theta_0) = \hat{\delta}(\theta_0) D(E_0/\hat{E}(\theta_0)) \quad ,$$

(7)

where

$$D(x) = \frac{sx}{s - 1 + x^s} \quad ,$$

(8)

$\delta_{ts}(E_0/\theta_0) =$ SEY,
E_0 and $\theta_0 =$ energy and incident angle of the electron,
$\hat{E} =$ energy at maximum D,
$\hat{\delta} =$ maximal SEY at θ_0,
Normal incident assumed.

3 Example and Numerical Results

3.1 Example A: PSR with a clean gap

Assume a parabolic proton line-density,

$$\lambda_p(z) = 6N_p s(1-s)/L \ , \tag{9}$$

and a weight function

$$W_e(z) = \frac{1}{3} + \frac{4}{3}s^2(3-2s) \ , \quad \text{so that } W_e(L/2) = 1 \ , \tag{10}$$

$s = z/L$, z = axial distance from the head of the bunch.

λ_p and W_e are shown in Figs. 1 and 2. Initial fraction of neutralization, $\chi \approx 3\%$ at $z \approx 0.5L$.

PSR parameter values: $\gamma = 1.85$, $a = 1.5$cm, $b = 5$cm, circumference $C = 90$m, $N_p = 2 \times 10^{13}$, $L/v = 260$ns, and $\nu_y = 2.1$.

The maximal electron bouncing frequency, at these parameter values, ≈ 160 MHz. Chose $C_d = 1.2 \times 10^5$/s.

$N = 520$, and $\Delta t = 0.5$ns.

Assume initially $Y_p(z,0) = 0.076\sin[(2\pi z)/(2.2\text{m})]$cm, and $dY_p/dt = 0$. When carried by the traveling proton beam, it corresponds to a wave of 126 MHz.
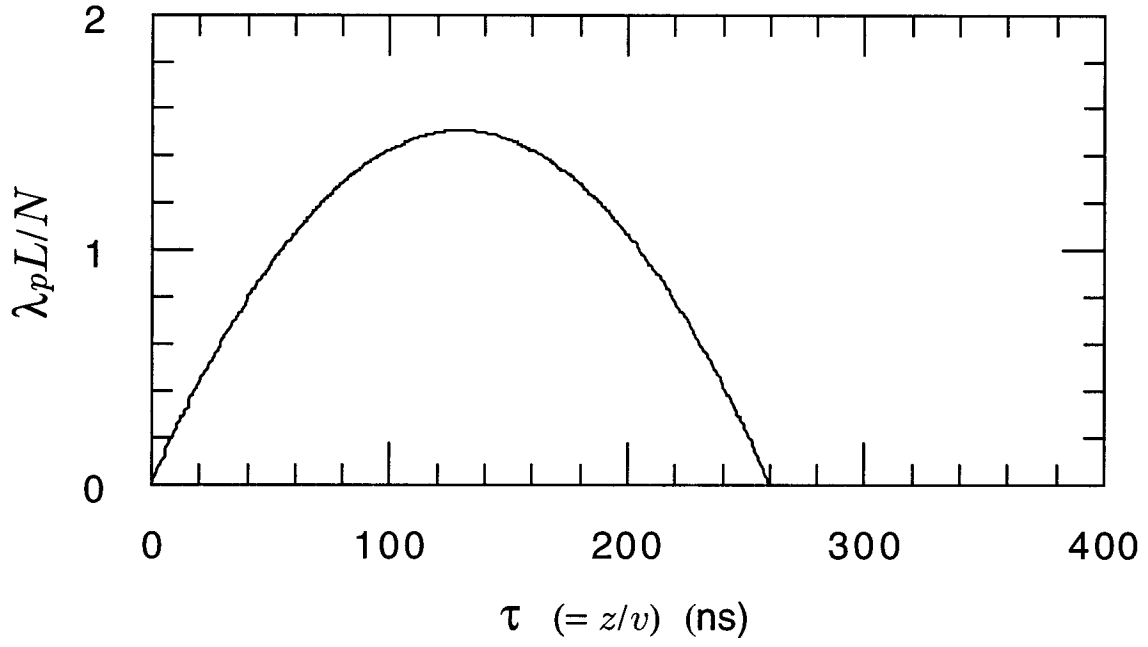
Fig. 1. The line densities used in the example shown here after normalized by N_p/L. The gap is between $\tau = 260$ns and 360ns.
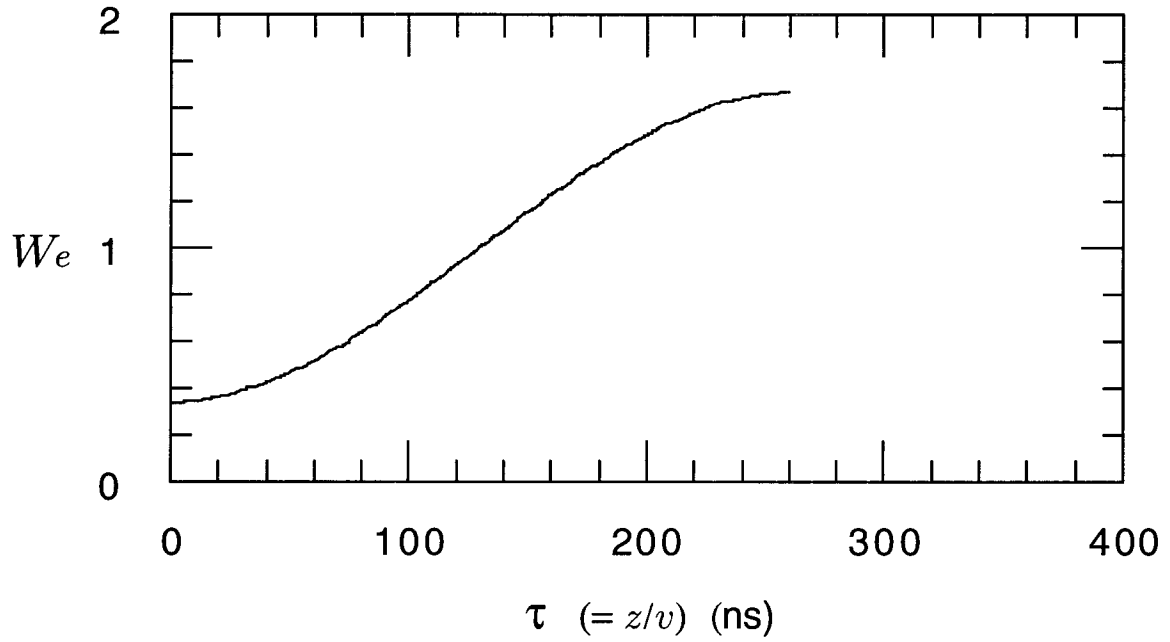

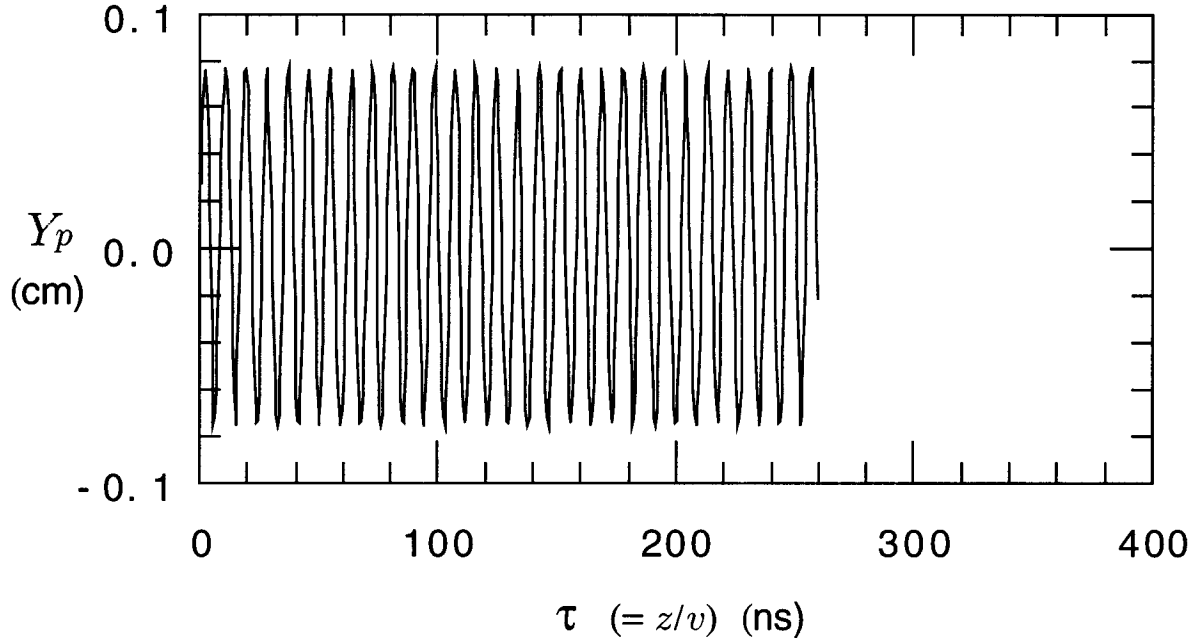Fig. 2. The weight function used in the example here.

Fig. 3. The initial perturbation on Y_p .

Initially, 17 core-electrons and 2 wall-electrons, per slice, are evenly distributed from wall to wall (-5cm to 5cm).

All electrons start at rest, and $a_e = 0.5$cm.

Initial charge assignment of macro-electrons,

$$c_{ec} = eN_p\chi/(n_{ec} + 0.5n_{ew}) , \quad \text{and} \quad c_{ew} = c_{ec}/2 .$$

For wall-electrons created at $t > 0$: $c_{ew} = c_{ec}(t = 0)/2$.

Maximum number of macro-electrons per slice $= 21$.

Secondary Emission: $\hat{\delta} = 2.0, \quad \hat{E} = 295$eV.

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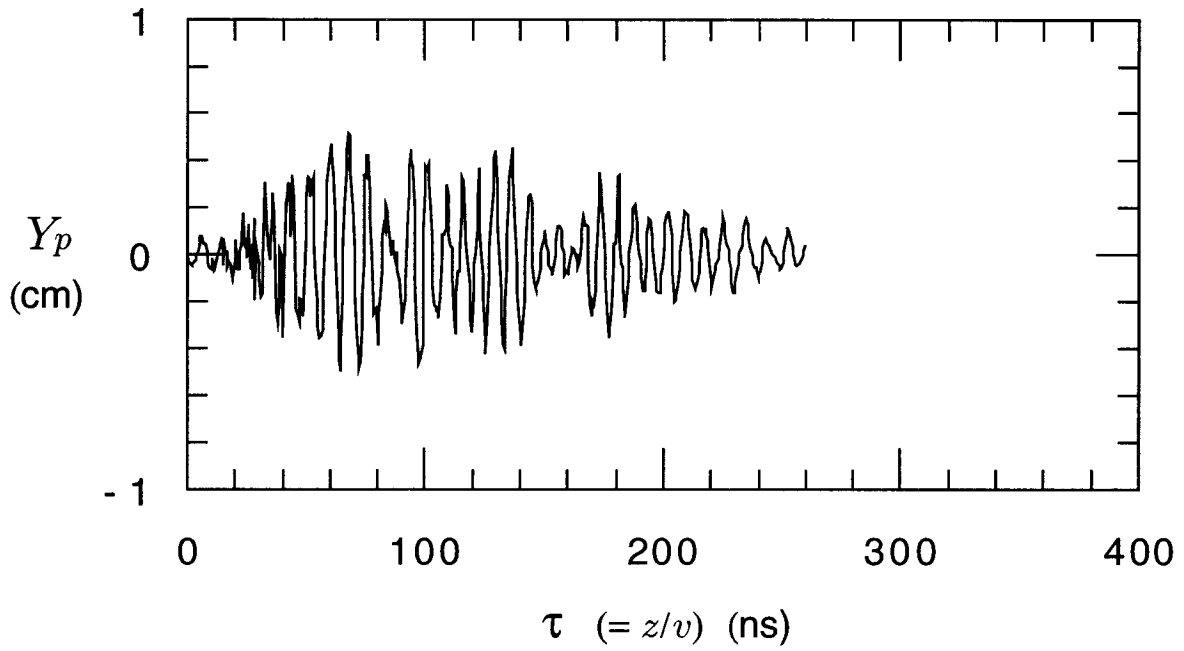
Fig. 4. The snap-shots of Y_p after tracking for a time interval equivalent to that of 80 revolutions of protons in PSR ($\approx 28.8\mu s$)
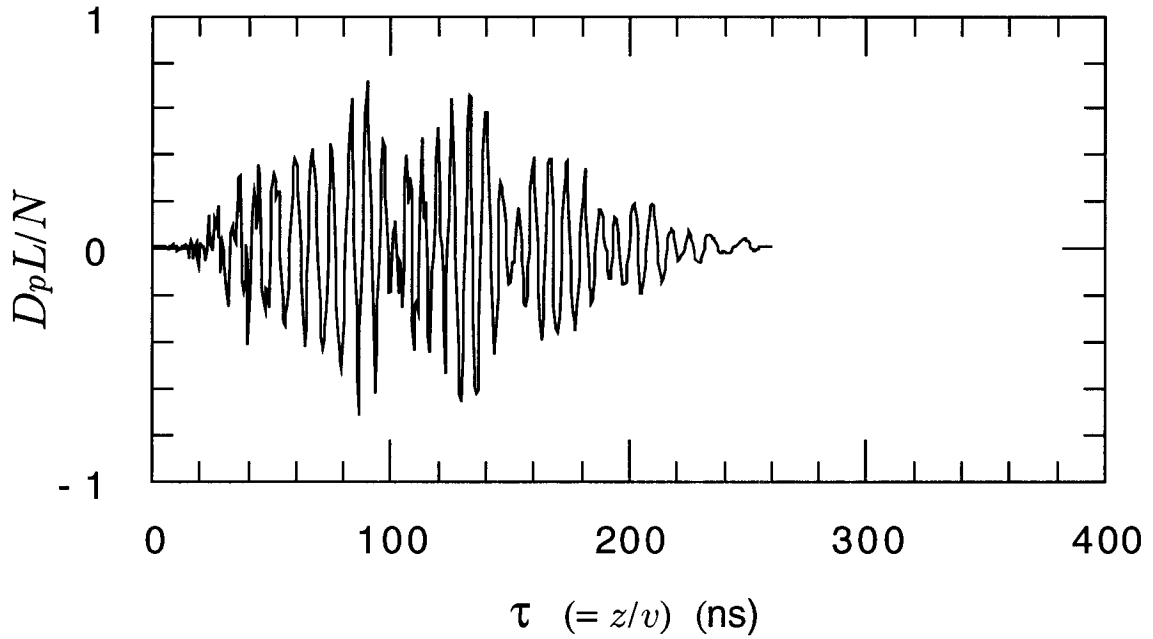

Fig. 5. The dipole moment density $D_p = \lambda_p Y_p$, after normalized by N_p/L, for the Y_p shown in Fig. 4.
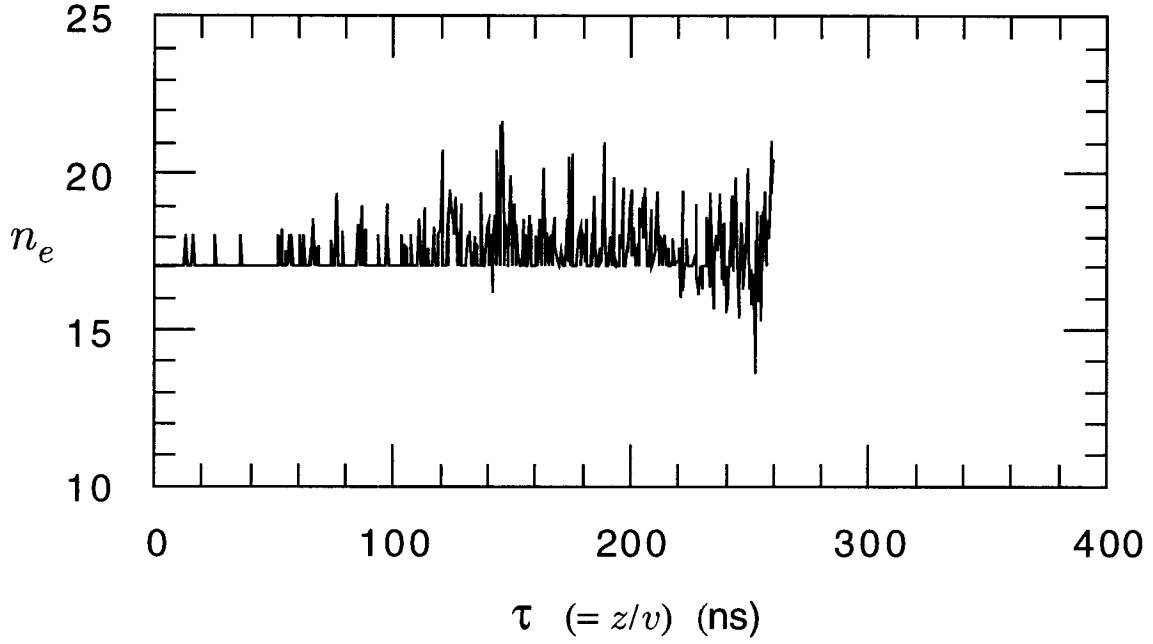
Fig. 6. The electron density n_e after tracking for 80 revolutions of protons in PSR ($\approx 28.8\mu s$)
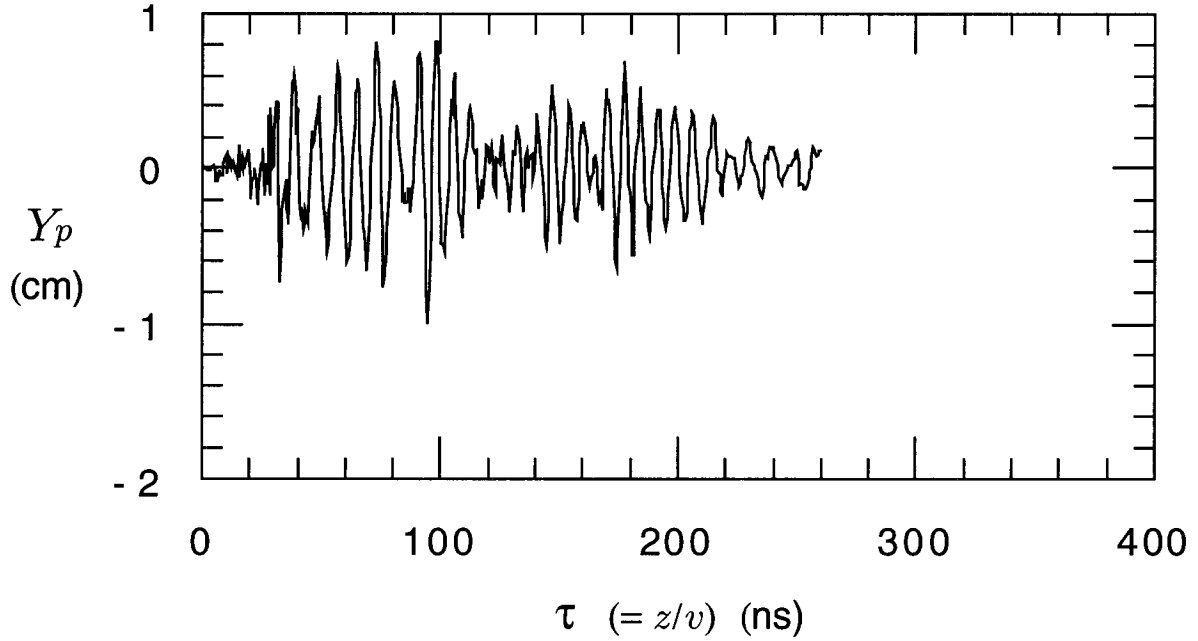

Fig. 7. The snap-shots of Y_p after tracking for a time interval equivalent to that of 160 revolutions of protons in PSR ($\approx 57.6\mu s$)
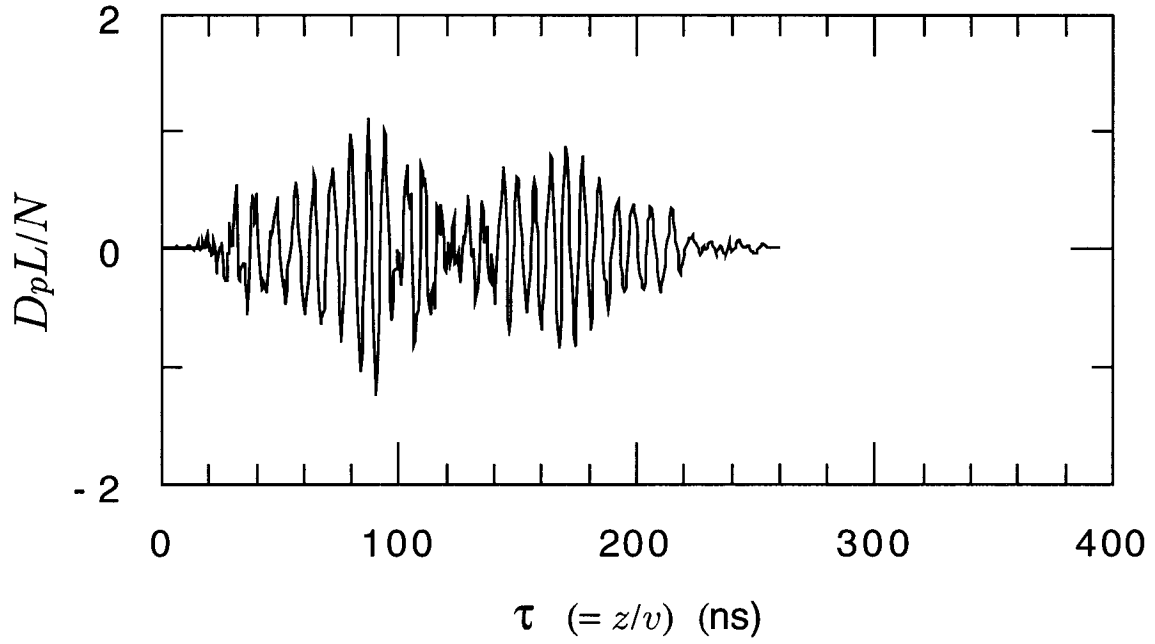
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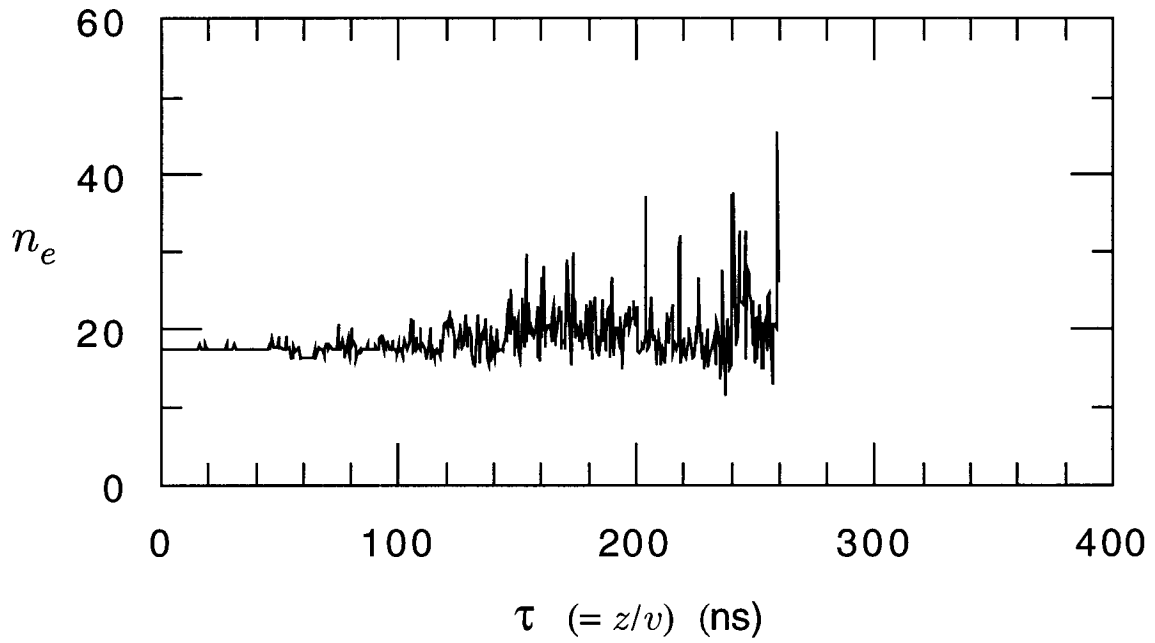Fig. 8. The dipole moment density $D_p = \lambda_p Y_p$, after normalized by N_p/L, for the Y_p shown in Fig. 7.

Fig. 9. The electron density n_e after tracking for 160 revolutions of protons in PSR ($\approx 57.6\mu s$)

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3.2 Example B: Multiplication threshold

Use the same initial conditions as in Example A (small perturbation).

Vary the beam intensity and check every time step in first turn to look for possible electron multiplication.

Observed multiplication at the tail of the bunch when $N_p \geq 1.3 \times 10^{13}$.
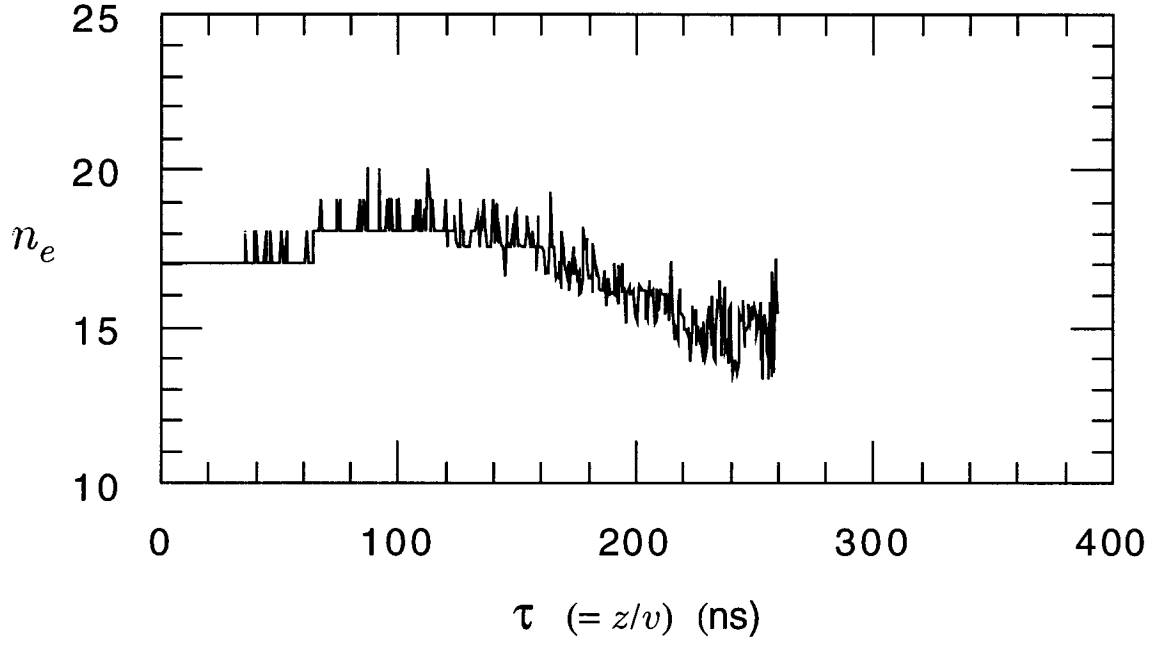
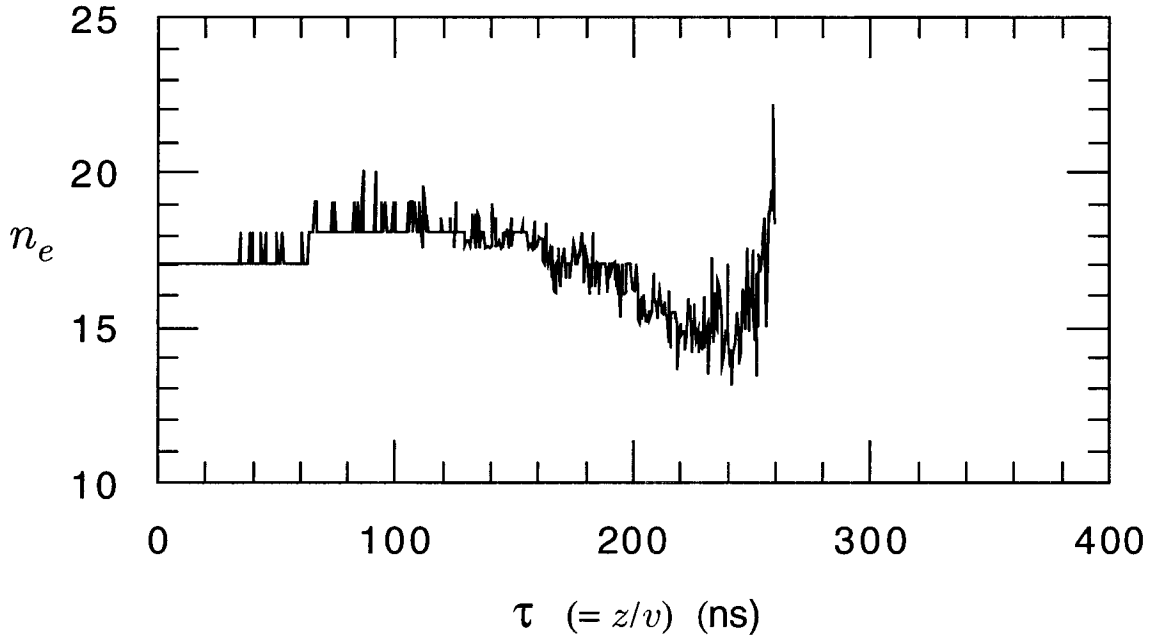Fig. 10. The electron density n_e after tracking for 128 time steps (≈ 64 ns). $N_p \approx 9 \times 10^{12}$.

Fig. 11. The electron density n_e after tracking for 128 time steps (≈ 64 ns). $N_p \approx 1.45 \times 10^{13}$.

3.3 General Results

- It takes only a few percent neutralization for the $e\text{-}p$ instability to develop in PSR. Computed short growth time consistent with observations.

- An empty gap does not always ensure the beam stability. Multi-turn trapping of electrons is not a necessary condition for instability.

- Roughly, the wavelength (or frequency) of the $e\text{-}p$ oscillation $\propto \sqrt{\lambda_p}$. Wide frequency spectrum for non-uniform λ_p.

- The $e\text{-}p$ instability grows both in time and space.

- For stainless steel SEY of $\hat{\delta} = 2.0$ at $\hat{E} = 295\text{eV}$, electron multipacting initially occurs only in the tail of the proton bunch. Appreciable multipacting occurs in the middle and the later part of the bunch after proton oscillation has grown to large amplitude ($> 0.5\text{cm}$).

4 Conclusions

- We have studied the *e-p* instability in a long proton bunch by solving the equation of motion for the centroid of the proton beam and the equations of motion for macro-electrons.

- The simulation here covers the production of secondary electrons on the beam pipe.

- The results here are consistent and are qualitatively in good agreement with the earlier simulations using the centroid model and experimental observations.

- It takes only a few percent of neutralization for the *e-p* instability to develop in PSR. Computed short growth time consistent with observations.

- An empty gap does not always ensure the beam stability. Multi-turn trapping of electrons is not a necessary condition for instability.

- For stainless steel SEY of $\hat{\delta} = 2.0$ at $\hat{E} = 295\text{eV}$, electron multipacting initially occurs only in the tail of the proton bunch. Threshold for multiplication at PSR parameter values is about $N_p \geq 1.3 \times 10^{13}$. Appreciable multipacting occurs in the middle and the later part of the bunch after proton oscillation has grown to large amplitude ($> 0.5\text{cm}$).